

Sudden transition from naked atom decay to dressed atom decay

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The studies on quantum open system play key roles not only in fundamental problems in quantum mechanics but also in quantum computing and information processes. Here we propose a scheme to use a one dimensional coupling cavity array (CCA) as an artificial electromagnetic environment of a two-level atom. For a finite length of CCA, we find that after a turning time the population of excited state deviates suddenly from the exponential decay. We show that physically this phenomena corresponds to a transition from a naked atom decay to a dressed state decay. We hope that our finding will promote the studies on quantum system with a finite size environment.

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I. INTRODUCTION

In nature there does not exist a completely closed quantum system. Any quantum system must interact with environment, and the environment in most cases is very complex and often not precisely specified, which makes it extremely difficult to study such an open system. One obvious reason for us to study an open system is that the knowledge on the system's state can not be obtained without the interaction between the measurement apparatus and the system. In addition, to control a quantum system to implement some quantum computing and information processes, we have to fight against the decoherence and dissipation induced by the environment [1].

Our aim is to study the effect of an environment with finite size. We model an environment with a one-dimensional coupling cavity array, which may be realized in experiments by coupled superconducting transmission line resonators or defect resonators in photonic crystals [2–5]. In literature, a one-dimensional coupling cavity array with a two-level atom in is first introduced to study single photon scattering problem [6]. Then many variants of this model are studied in recent years, for example, a super cavity can be formed by embedding two atoms to control the tunneling or directly decreasing two tunneling strengths [7, 8]. In addition, when putting an excited atom in the middle cavity of the CCA, the atom finally will evolve into two bound states and illustrate the oscillation behavior [9, 10].

Here we study the spontaneous decay of a two-level atom located in one node of a one dimensional (1D) coupling cavity array (CCA). For the atom, the 1D CCA can be regarded as a 1D electromagnetic environment. Because the length of the cavity array can be controlled in experiments, we have the opportunity to study the effect of an environment with finite size. For an environment with finite size, we may expect the spontaneous decay of

the atom will not be exponential one [11–15]. We find that there is a sudden deviation of the exponential decay at some turning time. We theoretically explain this phenomena as the transition from a naked atom decay to a dressed atom decay, and numerical results confirm this underlying mechanism.

The remainder of the paper is organized as follows. In Sec. 2, the theoretical model of a two-level atom in a 1D CCA is introduced as a microscopic model of an open quantum system. In Sec. 3, we analyze the condition for the exponential decay in our system, and gives the analytical result on the decay rate. In Sec. 4, we give the numerical results on the atom decay for the CCA with finite length, and find that the deviation of the exponential decay is a sudden transition at some turning time. We give a physical explanation why this happens and predict the turning time. In Sec. 5, after the turning time, we gives the results based on the dressed atom decay, which is confirmed by the numerical results. Finally we will give some discussions and a brief summary.

II. MODEL: A TWO-LEVEL ATOM IN 1D CCA

We consider a two-level atom located in the n -th cavity of a 1D CCA with length N as shown in Fig. 1, where the atom is our quantum system and the CCA is its 1D electromagnetic environment.



FIG. 1. (Color online). Schematic configuration of our system. A two-level atom (filled blue circle) is in the n -th cavity of the CCA.

Under the rotating wave approximation the Hamiltonian of the system becomes

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I + \mathcal{H}_A, \quad (1)$$

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where the Hamiltonian of the CCA H_0 is

$$\mathcal{H}_0 = \omega_c \sum_{j=1}^N a_j^\dagger a_j - \eta \sum_{j=1}^N (a_j^\dagger a_{j-1} + \text{h.c.}) \quad (2)$$

with ω_c the frequency of each single-mode cavity, η the hopping strength, N the number of cavities in the CCA, and a_j^\dagger (a_j) the photon creation (annihilation) operator for the j -th single-mode cavity. Hereafter, we set $\eta = 1$ and $\omega_c = 0$; The Hamiltonian for the free two-level atom \mathcal{H}_a is

$$\mathcal{H}_a = \omega_a |e\rangle\langle e|, \quad (3)$$

where ω_a is the energy of the excited state, $|e\rangle$ ($|g\rangle$) is the excited (ground) state of the atom. The interaction between the atom the cavity \mathcal{H}_I is

$$\mathcal{H}_I = g (a_n^\dagger \sigma_- + a_n \sigma_+), \quad (4)$$

where g is the coupling the atom and the cavity, and σ^- (σ^+) is the atomic lowering (raising) operator.

Notice that the total photon number $n_p = \sum_{j=1}^N a_j^\dagger a_j$ in H_0 is conserved for the CCA. In the subspace of $n_p = 1$, the eigenstate and eigenvalue of the CCA is

$$|\varphi_k\rangle = \sqrt{\frac{2}{N+1}} \sum_{j=1}^N \sin(j\theta_k) a_j^\dagger |vac\rangle = b_k^\dagger |vac\rangle, \quad (5)$$

$$\omega_k = -2 \cos \theta_k, \quad (6)$$

where k is integer with $1 \leq k \leq N$, $\theta_k = \frac{k\pi}{N+1}$, b_k (b_k^\dagger) the the photon creation (annihilation) operator for the k -th mode and $|vac\rangle$ is the vacuum state of the cavity array.

Using Eqs. (5) and (6), we can rewrite the Hamiltonian as

$$\mathcal{H} = \sum_{k=1}^N \omega_k b_k^\dagger b_k + \sum_{k=1}^N g_k (b_k^\dagger \sigma_- + \text{h.c.}) + \omega_a |e\rangle\langle e| \quad (7)$$

with $g_k = \sqrt{\frac{2}{N+1}} \sin(n\theta_k)g$ being the coupling between the atom and the k -th mode and. Notice that this is a typical spin-boson model to study quantum open system, and here the energy spectrum ω_k of the bosonic environment and the coupling strength g_k are specified.

III. EXPONENTIAL DECAY OF THE NAKED ATOM STATE

Now we study the spontaneous decay of the two-level atom in our system. In other words, when the atom is initially prepared in the excited state and the CCA stays in its vacuum state, what is dynamics of the excitation probability of the atom? In this section, we will focus on studying the conditions for the exponential decay of the excited atom, and theoretically predict the decay rate.

Due to the excitation number is conserved in our system, the state at any time t can be expanded as

$$|\Psi(t)\rangle = [\alpha(t)\sigma_+ + \sum_{k=1}^N \beta_k(t)b_k^\dagger] |vac\rangle |g\rangle, \quad (8)$$

where $\alpha(t)$ and $\beta_k(t)$ represent the excitation probability amplitude for the atom state and the k -th mode at time t respectively.

Through the Schrödinger equation

$$i \frac{d}{dt} |\Psi(t)\rangle = \mathcal{H} |\Psi(t)\rangle, \quad (9)$$

we get the following set of equations about the coefficients

$$i\dot{\alpha}(t) = -i\omega_a \alpha(t) - i \sum_{k=1}^N g_k \beta_k(t), \quad (10)$$

$$i\dot{\beta}_k(t) = -i\omega_k \beta_k(t) - ig_k \alpha(t). \quad (11)$$

Making the transformation of parameters

$$\begin{cases} \tilde{\alpha}(t) = \alpha(t) e^{i\omega_a t} \\ \tilde{\beta}_k(t) = \beta_k(t) e^{i\omega_k t} \\ \Delta_k = \omega_k - \omega_a \end{cases} \quad (12)$$

we simplify the equations of parameters as

$$\dot{\tilde{\alpha}}(t) = -i \sum_{k=1}^N g_k \tilde{\beta}_k(t) e^{-i\Delta_k t} \quad (13)$$

$$\dot{\tilde{\beta}}_k(t) = -ig_k \tilde{\alpha}(t) e^{i\Delta_k t} \quad (14)$$

Integrate Eq. (14) then substitute it into Eq. (13), we obtain an equation only about $\tilde{\alpha}(t)$

$$\dot{\tilde{\alpha}}(t) = - \sum_{k=1}^N g_k^2 \int_0^t dt' \tilde{\alpha}(t') e^{-i\Delta_k(t-t')}. \quad (15)$$

The exponential decay can be achieved when the near resonant modes are closely spaced in frequency and share almost the same coupling strength with the atom, meanwhile the other modes far off resonance are of little importance in the decay. To meet up with the requirements, we can use a cavity array with length long enough and put the atom in a selected location. Then the decay rate can be analytically derived. In our model, it is

$$\Gamma = \frac{4g^2}{\sqrt{4 - \omega_0^2}} \quad (16)$$

with $\omega_0 = 2 \cos k_0 = \omega_a$ the energy of the resonant mode (see Appendix for the detailed calculation).

We now give some numerical results to support the theoretical analysis above. Figure 2(a) shows the modes condition around the resonant frequency. With the atom set in the 1984-th cavity (the antinode of the resonant mode), the above mentioned requirements are satisfied. Furthermore, in a short time the behavior of the atom state is a

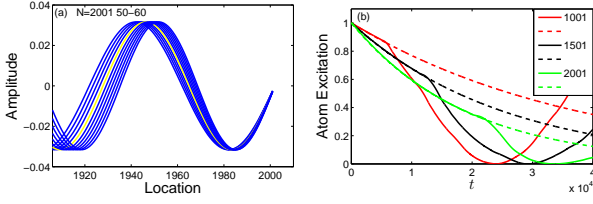


FIG. 2. (Color online). (a) The 50-th to 60-th modes of CCA with $N = 2001$. The yellow solid line stands for the mode (55-th) resonant with the atom while the other blue solid lines are for the modes nearby. (b) Time dependent atom state excitation for various lengths of CCA. The solid lines are the exact numerical results as for red $N = 1001, n = 992$, black $N = 1501, n = 1488$ and green $N = 2001, n = 1984$. The corresponding dash lines are exponential decay lines with the decay rate determined by Eq. (16). Here, $g = 0.0015$ and the atom is resonant with the 55-th mode in all situations.

truly well-defined exponential decay and with its decay rate well predicated by Eq. (16) (see Figure 2(b)).

But when time passes a specific point, the evolution dramatically deviates from the exponential line, and the change can be postponed by lengthening the cavity array. Next, we will explain the appearance of the turning point, and show that after the point a new physical process starts.

IV. DEVIATING FROM THE EXPONENTIAL DECAY

The key point for understanding the deviation is to clarify the behavior of the modes around the resonant frequency. We pick out the $N = 2001$ condition for example, Fig 3(a) reveals the evolution of the modes alongside with the atom state. Around the turning point, except the resonant mode other near-resonance modes almost all together oscillate to near zero. Fig 3(b) gives a more clear view about this pattern. This is the main reason to cause the deviation since the resonant mode becomes predominate over other near resonant modes at the turning point, which violates the requirements for the exponential decay of a naked atom.

Now we turn to give analytical analysis about the appearance of this pattern. As the atom state exponentially decay with time, its excitation probability amplitude is

$$\tilde{\alpha}(t) = e^{-\Gamma t/2}, \quad (17)$$

Then the k -th mode's excitation probability amplitude can be determined through Eq. (14)

$$\begin{aligned} \tilde{\beta}_k(t) &= -ig_k \int_0^t dt' e^{(i\Delta_k - \Gamma/2)t'} \\ &= \frac{-ig_k}{i\omega_k - \Gamma/2} [e^{(i\Delta_k - \Gamma/2)t} - 1], \end{aligned} \quad (18)$$

while its probability is

$$M_k(t) = |\tilde{\beta}_k(t)|^2 = A_k(e^{-\Gamma t} - 2e^{-\Gamma t/2} \cos \Delta_k t + 1), \quad (19)$$

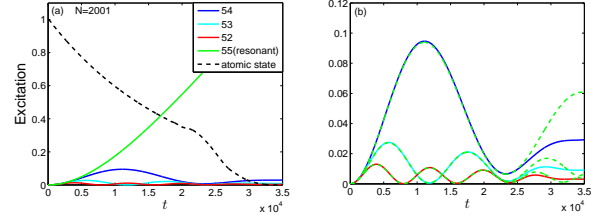


FIG. 3. (Color online). (a) Black dash line is the atom excitation and solid lines stand for various modes. (b) Only the near-resonance modes, for the solid lines each color represents the same mode as in (a), the yellow dashed line stands for the results calculate from Eq. (19). To simplify the figure, we only show some blue tuned near-resonance modes while not including the red tuned ones, their behavior actually are very much alike.

with $A_k = \frac{g_k^2}{\Gamma^2/4 + \omega_k^2}$.

Eq. (19) can perfectly describe the evolution of the modes during the time when the atomic state obeys the exponential decay as shown in Fig 3(b). For Δ_k , it satisfies the following equation $\Delta_k \approx (k - k_0)\Delta_1$ (see Appendix for proof), here we define $\Delta_1 = \omega_{k_0+1} - \omega_{k_0}$. To determine the minimum of M_k , we calculate its derivative

$$\dot{M}_k(t) = A_k e^{-\Gamma t/2} (-\Gamma e^{-\Gamma t/2} + \Gamma \cos \Delta_k t + 2\Delta_k \sin \Delta_k t). \quad (20)$$

For $\Delta_k \gg \Gamma$, only the third term in the right-hand side of Eq. (20) is predominant. Then minimums appear near

$$t_k = \frac{2\pi l}{\Delta_k} \approx \frac{2\pi l}{|k - k_0|\Delta_1} \quad (21)$$

as l is positive integer.

With Eq. (21) we can see when the excitation of the nearest mode (mode $k_0 \pm 1$) reach the first minimum, the other near-resonance modes also reach its minimum (though they have already oscillated several times). Then the time of the turning point t_c is totally dependent on the energy difference between the resonant and nearest near-resonance mode

$$t_c = \frac{2\pi}{\Delta_1} \quad (22)$$

And this is well consistent with the numerical results. When lengthening the cavity array, Δ_1 decreases and leads to the postpone the turning point as we mentioned before. For the extreme case, a 1D CCA with infinite length $\Delta_1 \rightarrow 0$ leads to $t_c \rightarrow \infty$. So this deviation phenomenon is actually due to the finite size of the environment.

V. DECAY OF THE DRESSED ATOM

Around the turning point, the effect of the resonant mode will be predominant since the other near-resonance modes are almost empty without any photon.

We conclude that the atom and the resonant mode will be strongly coupled to form a subsystem, the dressed atom [18]. So for the next stage of time, it is the decay of the dressed atom, a physical process totally different with before.

We can introduce a general master equation to describe the evolution of the dressed atom [18]

$$\frac{d}{dt}\rho = -i[H_d, \rho] - \frac{\Gamma}{2}(\sigma^+\sigma^-\rho_d + \rho\sigma^+\sigma^-) + \Gamma\sigma^-\rho\sigma^+ \quad (23)$$

with

$$H_d = \frac{1}{2}\omega_0\sigma_z + \omega_0 b_r^\dagger b_r + g_r(b_r^\dagger\sigma^- + h.c.) \quad (24)$$

the Hamiltonian of the dressed atom, $g_r (= \sqrt{\frac{2}{N+1}}g)$ the coupling strength between the atom and the resonant mode, ρ the density matrix of the dressed atom and Γ the decay rate of the subsystem (same as the the spontaneous emission rate of the atom).

Under the secular approximation, we can project the master equation over the dressed states

$$\frac{d}{dt}\rho_{ii} = -\frac{\Gamma}{2}\rho_{ii} \quad (25)$$

$$\frac{d}{dt}\rho_{12} = (-2ig_r - \frac{\Gamma}{2})\rho_{12} \quad (26)$$

with $i = 1, 2$, $\rho_{ij} = \langle i|\rho|j\rangle$, $|1\rangle = \frac{1}{\sqrt{2}}(|vac\rangle|e\rangle + |1_{k_0}\rangle|g\rangle)$ and $|2\rangle = \frac{1}{\sqrt{2}}(|vac\rangle|e\rangle - |1_{k_0}\rangle|g\rangle)$ the dressed states.

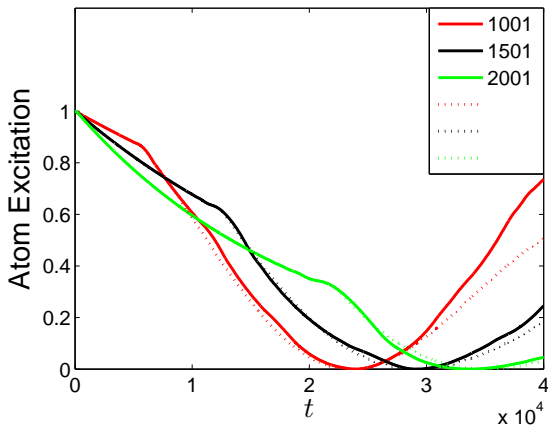


FIG. 4. (Color online). Atom excitation as a function of time. Solid lines represent exact numerical results with different color stands for different cavity array length. The dotted lines are obtained through Eq. (28). All the parameters are the same as before.

For the atom excitation

$$\rho_{ee}(t) = \frac{1}{2}[\rho_{11}(t) + \rho_{22}(t) + \rho_{12}(t) + \rho_{21}(t)] \quad (27)$$

Substitute the results obtained through Eq. (25) and Eq. (26) into Eq. (27), the evolution of the atom state is determined as

$$\rho_{ee}(t) = \frac{1}{2}e^{-\frac{\Gamma}{2}(t-t_c)}[\rho_{11}(t_c) + \rho_{22}(t_c) + \rho_{12}(t_c)e^{-2ig_r(t-t_c)} + \rho_{21}(t_c)e^{2ig_r(t-t_c)}] \quad (28)$$

with $\rho_{ee}(t) = \langle e|\langle vac|\rho(t)|vac\rangle|e\rangle$.

Fig 4 shows that after turning point, the dressed atom decay theory can describe the system evolution quite accurately. But after the time point when the atom state is empty, the dressed atom system no longer exists, the evolution becomes quite chaotic and unpredictable.

In conclusion, we study the atom decay within the 1D CCA frame. By selecting appropriate parameters, we achieve the well-known exponential decay of the atom state, and theoretically give the decay rate. Meanwhile the exponential decay only last a limited time in this model, the evolution deviates after a time point we name t_c . This is due to the near-resonance modes almost become empty together around t_c , and we theoretically prove the existence of t_c in our model. After t_c , the resonant mode is strongly coupled with the atom to form the dressed atom subsystem, and the dressed atom decay theory can describe the evolution quite accurately. Finally when the atom evolves into its ground state, the dressed atom subsystem no longer exists, and the evolution after is quite chaotic and beyond our understanding now. We hope our work can enlighten the study of the transition between the atom state exponential decay and dressed atom decay, and help to have a better understanding about the dynamics of quantum open system with a finite size environment.

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APPENDIX: HOW MANY MODES ARE ENOUGH?

Throughout the paper, we emphases that only modes around the resonant frequency are important in the system evolution while the far off-resonance modes can be ignored (this is also the key point in the Weisskopf-Wigner approximation). Then the question appears immediately, how to define near-resonance and far off-resonance or in another way say how many modes is actually enough? We give some numerical results to show intuitively that the number is quite small.

For the cavity array with $N = 2001$, Fig 5 reveals that the system can be described quite accurately using only

5 modes and perfectly with 11 modes. Compared with the total 2001 modes in the system, this is a very small number.

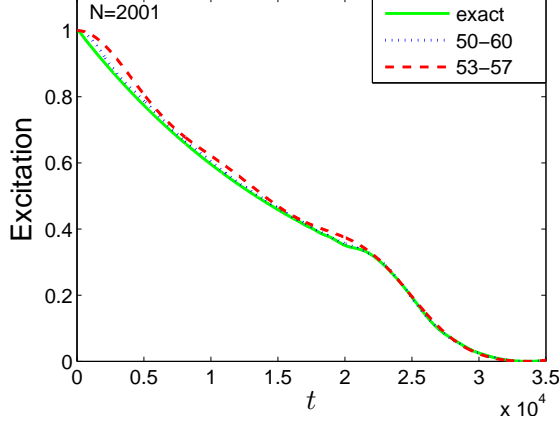


FIG. 5. (Color online). Time dependent atom excitation. The green solid line is the exact results including all the modes, the dotted blue line including the 50-th to 60-th modes and the red dashed line including the 53-th to 57-th modes. Here the other parameters are the same as before.

APPENDIX: CALCULATION OF THE DECAY RATE

As mentioned above, for the modes around the resonant frequency the coupling strength $g_k \approx g_r = \sqrt{\frac{2}{N+1}}g$, then we can rewrite Eq. (15) as

$$\dot{\tilde{\alpha}}(t) = -\frac{2g^2}{N+1} \sum_{k=1}^N \int_0^t dt' \tilde{\alpha}(t') e^{-i\Delta_k(t-t')}. \quad (29)$$

After replacing the summation over k with an integration as $dk = \frac{\pi}{N+1}$, we get

$$\dot{\tilde{\alpha}}(t) = -\frac{2g^2}{\pi} \int_0^\pi dk \int_0^t dt' \tilde{\alpha}(t') e^{-i\Delta_k(t-t')}. \quad (30)$$

By replacing the integral variable k with frequency ω since $\omega = 2 \cos k$, we have

$$\dot{\tilde{\alpha}}(t) = -\frac{2g^2}{\pi} \int_0^t dt' \int_{-2}^2 d\omega \frac{1}{\sqrt{4-\omega^2}} \tilde{\alpha}(t') e^{i(\omega_0-\omega)(t-t')} \quad (31)$$

Adopting the Weisskopf-Wigner approximation [16, 17], we get

$$\dot{\tilde{\alpha}}(t) = -\tilde{\alpha}(t) \int_0^\infty d\tau \int_{-2}^2 d\omega \frac{2g^2}{\pi\sqrt{4-\omega^2}} e^{i(\omega_0-\omega)\tau} \quad (32)$$

with $\tau = t - t'$. Since

$$\int_0^\infty d\tau e^{-i(\omega-\omega_0)\tau} = \pi\delta(\omega-\omega_0) - iP\left(\frac{1}{\omega-\omega_0}\right) \quad (33)$$

where P represents the Cauchy principal part which leads to the well-known lamb shift. Since the lamb shift has nothing to do with decay, we neglect it here and obtain

$$\dot{\tilde{\alpha}}(t) = -\Gamma \tilde{\alpha}(t) \quad (34)$$

with Γ the atom spontaneous emission rate

$$\Gamma = \frac{4g^2}{\sqrt{4-\omega_0^2}} \quad (35)$$

APPENDIX: PROOF OF $\Delta_k \approx (k - k_0)\Delta_1$

From Eq. (21) and Eq. (6), we have

$$\Delta_k = 2\left(\cos \frac{k\pi}{N+1} - \cos \frac{k_0\pi}{N+1}\right). \quad (36)$$

Using the sum and difference formulas of the trigonometric functions, we rewrite it as

$$\Delta_k = -4 \sin \frac{m\pi}{2(N+1)} \sin \frac{2k_0+m}{2(N+1)}, \quad (37)$$

where $m = k - k_0$. Since we only consider the modes around the resonant frequency, $m \ll N$ and $m \ll k_0$. Then Eq. (37) can be simplified as

$$\Delta_k \approx -4 \frac{m\pi}{2(N+1)} \sin \frac{2k_0+m}{2(N+1)} \approx (k - k_0)\Delta_1. \quad (38)$$

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- [1] H. P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, 2002).
 - [2] L. Zhou, Y. B. Gao, Z. Song, and C. P. Sun, Phys. Rev. A **77**, 013831 (2008).
 - [3] A. D. Greentree, C. Tahan, J. H. Cole, and L. C. L. Hollenberg Nature Phys. **2**, 856 (2006).

- [4] A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R. S. Huang, J. Majer, S. Kumar, S. M. Girvin, and R. J. Schoelkopf, Nature **431**, 162 (2004).
- [5] M. Mariantoni, F. Deppe, A. Marx, R. Gross, F. K. Wilhelm, and E. Solano, Phys. Rev. B **78**, 104508 (2008).
- [6] L. Zhou, Z. R. Gong, Y. X. Liu, C. P. Sun, and F. Nori, Phys. Rev. Lett. **101**, 100501 (2008).

- [7] Z. R. Gong, H. Ian, L. Zhou, and C. P. Sun, Phys. Rev. A **78**, 053806 (2008).
- [8] W. Zhu, Z. H. Wang, and D. L. Zhou, Phys. Rev. A **90**, 043828 (2014).
- [9] Q. J. Tong, J. H. An, H. G. Luo, and C. H. Oh Phys. Rev. B **84**, 174301 (2011).
- [10] F. Lombardo, F. Ciccarello, and G. M. Palma, Phys. Rev. A **89**, 053826 (2014).
- [11] H. Gieβen, J. D. Berger, G. Mohs, and P. Meystre, Phys. Rev. A **53**, 2816 (1996).
- [12] J. M. Zhang, M. Haque, arXiv:1404.4280.
- [13] G. C. Stey and R. W. Gibberd, Physica **60**, 1(1972).
- [14] M. Ligare and R. Oliveri, Am. J. Phys. **70**, 58 (2002).
- [15] Y. Guo, Z. H. Wang and D. L. Zhou, Phys. J. D **68**, 110(2014).
- [16] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge university Press, Cambridge, England, 1997).
- [17] L. Novotny and M. B. Hecht, *Principles of Nano-Optics* (Cambridge university Press, Cambridge, England, 2006).
- [18] C. Cohen-Tannoudji, J. Dupont-Roc and G. Grynberg, *Atom-Photon Interactions* (WILEY-VCH Verlag GmbH Co. KGaA, Weinheim, Germany, 2004).